

DSP

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محاضرة [4]

Shifting property :

خاصية ①

→ Delay ✓ هو إزاحة إلى الخلف
→ Advance

- Find Z.T for $x(n-1)$

Delay

$$x(n) \xrightarrow{Z.T} Z[x(n)] = X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = x(0) + x(1)z^{-1} + \dots$$

$$Z[x(n-1)] = \sum_{n=-\infty}^{\infty} x(n-1) z^{-n}$$

$$= x(-1) + x(0)z^{-1} + x(1)z^{-2} + x(2)z^{-3} + \dots$$

$$= x(-1) + z^{-1} [x(0) + x(1)z^{-1} + x(2)z^{-2} + \dots]$$

$$= x(-1) + z^{-1} X(z) = z^{-1} X(z)$$

$$Z[x(n-2)] = \sum_{n=-\infty}^{\infty} x(n-2) z^{-n}$$

$$= x(-2) + x(-1)z^{-1} + x(0)z^{-2} + x(1)z^{-3} + \dots$$

$$= x(-2) + x(-1)z^{-1} + z^{-2} [x(0) + x(1)z^{-1} + x(2)z^{-2} + \dots]$$

assume $x(n) = 0, n < 0$

$$x(-1), x(-2), x(-3), \dots = 0$$

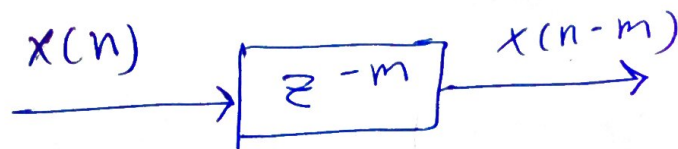
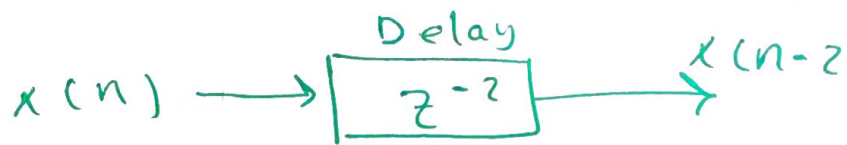
$$Z[x(n-2)] = \cancel{x(-2)} + \cancel{x(-1)z^{-1}} + z^{-2} X(z)$$

$$= z^{-2} X(z)$$

$$Z[x(n-m)] = z^{-m} X(z)$$

①

in control design (generally)



* Advance Shift :-

Find Z.T for $x(n+1)$

$$Z[x(n+1)] = \sum_{n=0}^{\infty} x(n+1) z^{-n}$$

$$= x(1) + x(2)z^{-1} + x(3)z^{-2} + x(4)z^{-3} + \dots$$

$$= z z^{-1} [x(1) + x(2)z^{-1} + x(3)z^{-2} + x(4)z^{-3} + \dots]$$

$$= z [x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3} + \dots]$$

$$= z \left[\underbrace{x(0) + \dots}_{\text{فرد کما هو}} - x(0) \right]$$

$$= z [x(z) - x(0)]$$

$$= z x(z) - z x(0)$$

$$* Z[x(n+1)] = z x(z) - z x(0)$$

$$- Z [x(n+2)] = \sum_{n=0}^{\infty} x(n+2) z^{-n}$$

$$= x(2) + x(3) z^{-1} + x(4) z^{-2} + \dots$$

$$= z^2 z^{-2} [x(2) + x(3) z^{-1} + \dots]$$

$$= z^2 [x(2) z^{-2} + x(3) z^{-3} + \dots]$$

مضروب
بـ z^2 $\Rightarrow$

$$x(0) - \cancel{x(0)} + x(1) z^{-1} - \cancel{x(1) z^{-1}}$$

$$= z^2 [x(0) + x(1) z^{-1} + x(2) z^{-2} + x(3) z^{-3} + \dots - x(0) - x(1) z^{-1}]$$

$$= z^2 [x(2) - x(0) - x(1) z^{-1}]$$

$$= z^2 x(2) - z^2 x(0) - z x(1)$$

in control design (generally



for difference eqn we will deal with delay ONLY

initial value

(2) $\lim_{z \rightarrow \infty}$

$$x(0) = x(n) \Big|_{n=0} = \lim_{z \rightarrow \infty} x(z)$$

- Final Value

(3) $\lim_{z \rightarrow 1}$

$$x(\infty) = x(n) \Big|_{n=\infty} = \lim_{z \rightarrow 1} (z-1) x(z)$$

$$= \lim_{z \rightarrow 1} (1 - z^{-1}) x(z) \quad \begin{matrix} \nearrow z-1 \\ \searrow z \end{matrix}$$

* Inverse Z.T :- $\boxed{Z^{-1}T}$

$$\begin{array}{ccc} x(n) & \xrightarrow{Z.T} & X(z) \\ \text{discrete} & \xleftarrow{Z^{-1}T} & \text{z-domain} \\ \text{time domain} & \text{Inverse Z.T} & \end{array}$$

there are two methods :-

$X(z)$ ① using PF (Partial Fraction) $\Rightarrow$ (closed form sol)

$X(0), X(1), X(2) \dots$ ② using LD (Long Division) $\Rightarrow$ (open form sol.)

Ex: $X(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$

Find $x(n)$

(closed form sol.)

Solution :-

$$X(z) = \frac{z^2}{z^2 - 1.5z + 0.5}$$

$$\downarrow$$

$$(z-1)(z-0.5)$$

$$X(z) = z \left[\frac{z}{(z-1)(z-0.5)} \right]$$

$$= z \left[\frac{A_1}{z-1} + \frac{A_2}{z-0.5} \right]$$

$$X(z) = z \left[\frac{2}{z-1} - \frac{1}{z-0.5} \right]$$

$$= \frac{2z}{z-1} - \frac{z}{z-0.5}$$

$$A_1 = \frac{1}{1-0.5} = 2$$

$$A_2 = \frac{0.5}{-0.5} = -1$$

$$Z^{-1}T \Rightarrow x(n) = 2u(n) - (0.5)^n u(n)$$

Ex: $X(z) = \frac{1}{(z-1)(z-0.3)} \Rightarrow$ use partial fractions

$$X(z) = \frac{A_1}{z-1} + \frac{A_2}{(z-0.3)}$$

$$X(z) = \frac{1.4 z z^{-1}}{z-1} - \frac{1.4 z z^{-1}}{(z-0.3)} \quad \begin{matrix} \downarrow \text{shift} \\ \end{matrix}$$

$$\left\{ \begin{array}{l} A_1 = \frac{1}{0.7} = 1.4 \\ A_2 = \frac{-1}{0.7} = -1.4 \end{array} \right.$$

$z^{-1} T \Rightarrow x(n) = 1.4 u(n-1) - 1.4 (0.3)^{n-1} u(n-1)$

at $n=0 \Rightarrow x(0) = 0$

OR $x(0) = \lim_{z \rightarrow \infty} \frac{1}{(z-1)(z-0.3)} = \frac{1}{\infty} = 0$

Ex: $X(z) = \frac{z^3}{(z-1)(z^2-z+1)}$

- ① find $x(0), x(1), x(2)$ using long Division method
- ② $x(n)$ using P.F.

Solution: - $(z-1)(z^2-z+1) = z^3 - z^2 + z - z^2 + z - 1 = z^3 - 2z^2 + 2z - 1$

$$X(z) = \frac{z^3}{z^3 - 2z^2 + 2z - 1}$$

$\Rightarrow$ Turn over

$$\begin{array}{r}
 1 + 2z^{-1} + 2z^{-2} \\
 \hline
 z^3 - 2z^2 + 2z - 1 \quad \boxed{\begin{array}{r} z^3 \\ -z^3 + 2z^2 + 2z - 1 \\ \hline 2z^2 - 2z + 1 \\ -2z^2 + 2z^2 + 2z - 1 \\ \hline 2z - 3 + 2z^{-1} \end{array}}
 \end{array}$$

$$X(z) = 1 + 2z^{-1} + 2z^{-2} + \dots$$

$$= X(0) + X(1)z^{-1} + X(2)z^{-2} + \dots$$

①

$$\begin{array}{l}
 X(0) = 1 \\
 X(1) = 2 \\
 X(2) = 2
 \end{array}$$

②

ممكن يقول هات الفقرة الأولى بالأسور الجزئية، مل وعوض
وصات المطلوب .

using P F :

$$X(z) = \frac{z^3}{(z-1)(z^2-z+1)}$$

$$= z \left[\frac{z^2}{(z-1)(z^2-z+1)} \right]$$

$$= z \left[\frac{A}{z-1} + \frac{Bz+C}{z^2-z+1} \right]$$

$\Rightarrow$ Turn over

$$X(z) = z \left[\frac{A}{(z-1)} + \frac{1}{(z^2 - z + 1)} \right] \quad \left| \begin{array}{l} A = \frac{1}{1-1+1} = 1 \\ B = 0 \\ C = 1 \end{array} \right. \quad \left. \begin{array}{l} \text{poles} \\ \text{zeros} \end{array} \right.$$

$$= \frac{z}{z-1} + \frac{z}{z^2 - z + 1}$$

$$\sin(\omega n) \xrightarrow{z.T} \frac{z \sin(\omega)}{z^2 - 2z \cos(\omega) + 1}$$

$$\frac{z \sin(\omega)}{z^2 - z + 1} \iff \frac{z \sin \omega}{z^2 - 2z \cos(\omega) + 1}$$

$$2 \cos(\omega) = 1 \Rightarrow \cos \omega = 1/2$$

$$\omega = 60^\circ = \frac{\pi}{3} \text{ radian}$$

$$\sin \omega = \sin 60 = \sqrt{3}/2$$

$$\therefore X(z) = \left[\frac{z}{z-1} + \frac{z(\sqrt{3}/2)}{\sqrt{3}/2 (z^2 - z + 1)} \right] \quad \begin{array}{l} \swarrow \sin(\omega n) = \sin(\pi/3 n) \\ \searrow \end{array}$$

$$z^{-1}T \Rightarrow x(n) = u(n) + \frac{1}{\sqrt{3}/2} \sin\left(\frac{\pi}{3}n\right)$$

$$\text{Ex: } y(n) - y(n-1] + 0.25 y(n-2) = u(n)$$

for unit step input:

- ① Find $y(0), y(1), y(2)$ using Long Division method
- ② find the unit step response ($y(n)$) using P.F.

Solution:-

$$z.T \Rightarrow Y(z) - z^{-1}Y(z) + 0.25z^{-2}Y(z) = \frac{z}{z-1}$$

$$Y(z)(1 - z^{-1} + 0.25z^{-2}) = \frac{z}{z-1}$$

$$Y(z) = \frac{z}{(z-1)(1 - z^{-1} + 0.25z^{-2})}$$

$$Y(z) = \frac{z^3}{(z-1)(z^2 - z + 0.25)}$$

① Using Long Division:-

$$Y(z) = \frac{z^3}{z^3 - 2z^2 + 1.75z - 0.25}$$

$$\begin{array}{r} z^3 - 2z^2 + 1.75z - 0.25 \overline{) 1 + 2z^{-1} + 2.75z^{-2} + \dots} \\ \underline{z^3 - 2z^2 + 1.75z - 0.25} \\ 2z^2 - 1.75z + 0.25 \\ \underline{-2z^2 + 4z - 2.5 + 0.5z^{-1}} \\ 2.75z - 2.25 + 0.5z^{-1} \end{array}$$

$$y(0) = y(n) \Big|_{n=0} = 1$$

$$y(1) = y(n) \Big|_{n=1} = 2$$

$$y(2) = y(n) \Big|_{n=2} = 2.75$$

2) Using PF

$$Y(z) = \frac{z^3}{(z-1)(z^2-z+0.75)}$$

$$= \frac{z^3}{(z-1)(z-0.5)^2}$$

using PF $\rightarrow$

$$= z \left[\frac{z^2}{(z-1)(z-0.5)^2} \right]$$

$$= z \left[\frac{A_1}{(z-1)} + \frac{A_2}{(z-0.5)^2} + \frac{A_3}{(z-0.5)} \right]$$

$$Y(z) = \left[\frac{4z}{(z-1)} - \frac{0.5z}{(z-0.5)^2} - \frac{3z}{z-0.5} \right]$$

$$Y(n) = \left[4u(n) - n(0.5)^n - 3(0.5)^n \right]$$

$$A_1 = \frac{1}{(0.5)^2}$$

$$= 4$$

$$A_2 = \frac{0.5^2}{0.5-1}$$

$$= -0.5$$

$\Rightarrow$ Turn over

to find A_3

Put $z=0$

$$0 = -A + 4A_2 - 7A_3$$

$$\Rightarrow A_3 = -3$$

$$\frac{z}{(z-0.5)^2} = \frac{z}{(0.5)^2 \left(\frac{z}{0.5} - 1 \right)^2} = \frac{0.5 z / 0.5}{0.25 \left(\frac{z}{0.5} - 1 \right)^2}$$

(9)

$$\text{صورة عامة } a^n n \xrightarrow{z.T} \frac{z}{z - \frac{z}{a}} \bigg|_{z = \frac{z}{a}} = \frac{z/a}{(z/a - 1)^2} = \frac{a z}{(z - a)^2}$$

$$\text{Final Value} \Rightarrow y(\infty) = 4$$

$$\text{Ex: } X(z) = \frac{4z}{z^2 + 0.1z + 0.2}, \text{ find } x(n)$$

$$\sin(\omega n) \xrightarrow{z.T} \frac{z \sin \omega}{z^2 - 2z \cos \omega + 1}$$

$$a^n \sin(\omega n) \xrightarrow{z.T} \frac{z/a \sin \omega}{(\frac{z}{a})^2 - 2(\frac{z}{a}) \cos \omega + 1} = \frac{a z \sin(\omega)}{z^2 - 2a z \cos \omega + a^2}$$

$$z^2 + 0.1z + 0.2 \quad \swarrow$$

$$\equiv z^2 - 2a z \cos \omega + a^2$$

$$a^2 = 0.2 \Rightarrow a = \sqrt{0.2}$$

$$0.1 = -2a \cos \omega$$

$$\Rightarrow \omega = 96.4^\circ$$

$$= 1.68 \text{ rad}$$

$$\sin \omega = 0.99$$

$$\therefore X(z) = \frac{4z(a \sin(\omega))}{a \sin(\omega) [z^2 + 0.1z + 0.2]}$$

$$z^{-1} T =$$

$$\Downarrow$$

$$x(n) = \frac{4}{a \sin(\omega)} a^n \sin(\omega n)$$

$$x(n) = 4 (\sqrt{0.2})^n \sin(1.68n)$$

$$\text{Report}$$

$$X(z) = \frac{z}{z^2 + 4}$$

$$\text{find } x(n)$$